Proposal of demonstration

of the Goldbach's conjecture

Rémy Aumeunier
Amateur

24 Juillet 2020

Abstract

The Goldbach's conjecture is the unproven mathematic assertion which states : All whole even number greater than 2 can be written as the sum of two prime numbers. It was formulated in 1742 by Christian Goldbach, it is one of the oldest unsolved problem of the theorie of number and mathematics. It shares with the Rieman's hypothesis and the conjecture of twins prime numbers, the number 8 of Hilberts's [1] In this document I propose a demonstration of the conjecture ,if I consider the set of all modulos $< \sqrt{2n}$ it becomes trivial to decompose $2n$ into sum of 2 prime numbers , thank you for attention.

1 Preamble

This presentation follows many exchanges on various moderated forums, French or English, they have made it possible to make this demonstration proposal as unambiguous as possible, while answering any questions, I thank the reviewers for their contributions.

2 Primality testing

In this proposal to demonstrate Goldbach's [2] conjecture I use a primality test based on two prerequisites.
-Any number composed at a prime factor less than its square root. Then $n = p \cdot q + r$ if $r = 0$, n is divisible by a prime number $< \sqrt{n}$.

$$n \in N, E_q = \{q \quad \text{prime} \quad \text{number} \mid q \leq \sqrt{n}\}$$

$$\text{si } \forall q \in \ E_q, (n) mod(q) \neq 0 \text{ then } n \text{ is a prime number.}$$

[1] At the Second International Congress of Mathematicians, held in Paris in August 1900, David Hilbert presented a list of problems that until then had kept mathematicians in failure.

[2] On June 7, 1742, the Prussian mathematician Christian Goldbach wrote a letter to the Swiss mathematician Leonhard Euler at the end of which he proposed the following conjecture: Any number strictly greater than 2 can be written as the sum of two prime numbers.

To understand the following demonstration, we must consider that each calculation $(n)mod(q)$ is a measure, and if all measures are different from zero then n is a prime number, because any compound number has a prime factor less than its square root.

$$R_n = \begin{pmatrix} (n)mod(2) = \{1\} \\ (n)mod(3) = \{1,2\} \\ (n)mod(5) = \{1,2,3,4\} \\ (n)mod(7) = \{1,2,3,4,5,6\} \\ \cdots \\ \cdots \\ (n)mod(q_{max}) = \{1,\cdots,(q_{max}-1)\} \end{pmatrix}$$

So if an integer to all these decompositions in the form of modulo constitute of value different from zero then, this integer is a prime number, provided of course to respect the definition domain $q_{max} \leq \sqrt{n}$.

3 Goldbach's conjecture

To demonstrate Goldbach's conjecture, I hypothesize that there is an even integer $2n$ >2 that is not decomposable into the sum of 2 prime numbers.

$$n \in N$$

$$E_p = \{p \quad \text{prime} \quad \text{number} \mid p < 2n\}$$
$$E_q = \{q \quad \text{prime} \quad \text{number} \mid q \leq \sqrt{2n}\}$$

So I have a list or set E_p with all the prime numbers eligible for decomposition into $2n$, and a second set E_q that will be used to test the result of the $(2n-p)$ decomposition. Then as :

$$(2n-p)mod(q) = (2n)mod(q) - (p)mod(q)$$

If $2n$ is not decomposable into the sum of 2 prime numbers then :

$$\forall p \in E_p, \exists q \in E_q, 2n \equiv (p)mod(q)$$

A numerical example with $2n = 300$ and $p = 41, \in E_p$ which does not decompose $2n$ into the sum of 2 prime numbers.

$2n = 300, \sqrt{300} = 17.3, E_q = \{2,3,5,7,11,13,17\}, E_p = \{2,3,5,7,...\mathbf{41},...283,293\}$

$$R_{2n=300} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathbf{6} \\ 3 \\ 1 \\ 11 \end{pmatrix} - 41 \begin{pmatrix} 1 \\ 2 \\ 1 \\ \mathbf{6} \\ 8 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} .. \\ .. \\ .. \\ \mathbf{0} \\ .. \\ .. \\ .. \end{pmatrix} = 7 \cdot 37 + \mathbf{0}$$

Here if I make the assumption that 300 is not decomposable then all prime numbers less than 300 share at least one value with $R_{2n=300}$. But this isn't possible, because I can create, fabricate or build a set with different values of 0 and of $(2n)mod(q)$ and this set will be a prime number, and decomposed $2n$ because, by construction, there won't be any zero, in their representations in the form of a modulo. So, all even number are decomposable in sum of 2 prime numbers , because I can create, fabricate or build such a set.

$$\{(p)mod(q)\} \neq \{(2n)mod(q)\}$$

3.1 Note

This notion can also be used to justify Legendre's conjecture or the conjecture of twin prime numbers [3]. I can create, fabricate or construct a R_p set with values different from 0 such as $(R_p - 2) \neq 0, \exists$, and this, regardless of the size of the R_p set.

4 Analytical demonstration

From the remains or decomposition of $(2n - p)$ modulo.

$$\begin{pmatrix} (2n-p)mod(2) \\ (2n-p)mod(3) \\ (2n-p)mod(5) \\ ... \\ (2n-p)mod(q_n) \end{pmatrix} = \begin{pmatrix} (2n)mod(2) - (p)mod(2) \\ (2n)mod(3) - (p)mod(3) \\ (2n)mod(5) - (p)mod(5) \\ ... \\ (2n)mod(q_n) - (p)mod(q_n) \end{pmatrix} = \begin{pmatrix} r_1 - (p)mod(2) \\ r_2 - (p)mod(3) \\ r_3 - (p)mod(5) \\ ... \\ r_n - (p)mod(q_n) \end{pmatrix}$$

Here I am assuming that there exists an even integer $2n$ which is not decomposable into a sum of 2 prime numbers. This assumption implies that all the prime numbers $< 2n$ are of the form $n \cdot q + r_n$, but this is not possible because the Lejeune-Dirichlet theorem shows that there is an infinity of prime numbers of the form $k \cdot p + q$, with p and q prime between them. So the theorem says there exists a prime number such that $p_n = k \cdot p + q$ with $q \notin \{r_1, r_2, r_3, ...r_x\}$ and this prime number will decompose $2n$ because there will be no zero in the modulo representation of the result of $(2n - p)$.

5 Mode of operation, decomposition of 2n

After apprehending how and why $2n$ can be decomposed. I propose to create, fabricate or construct such a set, among all possible solutions, one of the least controversial methods, is to notice that in the finite countable set of modulos of $2n$, and that $\forall\ 2n$, there is always one or more primes $\in E_q$ that will be absent

[3] The Legendre conjecture and the twin prime number conjecture are among the four problems about prime numbers that Edmund Landau presented at the 1912 International Congress of Mathematicians in Cambridge. In 2020, they are unsolved.

from this set, because I can't create a set with, all the primes $< \sqrt{2n}$, and one of these absent primes will decompose $2n$, because there will be no zero present, in the modulo representation of $(2n - p_{abs})$. So any even number is decomposable into the sum of 2 primes.

6 Note from the author

Frustrating, isn't it? I can always decompose $2n$ into the sum of 2 primes, because there is no even integer that has a modulo decomposition where all $< \sqrt{2n}$ primes are present. Yes but no, it turns out that I can also build a whole arithmetic from these sets, which remains to be defined, unless I am mistaken of course. In any case, thank you for your attention.

References

[1] Goldbach's conjecture .

[2] Legendre's conjecture .

[3] Conjecture of prime numbers twins, triplets, quadruplets.

[4] distribution asymptotique des nombres premiers

[5] A very personal concept (the common denominator of form)